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*Published in:*  
I E E E Transactions on Magnetics

*Link to article, DOI:*  
[10.1109/20.133770](https://doi.org/10.1109/20.133770)

*Publication date:*  
1991

*Document Version*  
Publisher's PDF, also known as Version of record

[Link back to DTU Orbit](#)

*Citation (APA):*  
Holst, T., Hansen, J. B., Grønbech-Jensen, N., & Blackburn, J. A. (1991). Phase-locked Josephson soliton oscillators. *I E E E Transactions on Magnetics*, 27(2), 2704-2707. <https://doi.org/10.1109/20.133770>

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## PHASE-LOCKED JOSEPHSON SOLITON OSCILLATORS

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**Abstract**

Arrays of coherent Josephson oscillators offer larger microwave power and narrower linewidth than single junctions. We present detailed experimental characterization of the phase-locking both at dc and at microwave frequencies for two closely spaced Josephson soliton (fluxon) oscillators. In the phase-locked state the radiated microwave power exhibited an effective gain. With one common bias source a frequency tunability of the phase-locked oscillators up to 7 % at 10 GHz was observed. The interacting soliton oscillators were modelled by two inductively coupled non-linear transmission lines.

**Introduction**

Phase-locking of Josephson junctions has been studied extensively for systems of *small* junctions, i.e. for junction dimensions smaller than  $\lambda_J$ , the Josephson penetration depth.<sup>1</sup> A Josephson transmission line (JTL), i.e. a long and narrow, quasi-one-dimensional, Josephson tunnel junction of length  $l$  and width  $w$ , supports self-resonant soliton (fluxon) modes.<sup>2</sup> We have studied phase-locking between two JTL soliton oscillators both experimentally and by means of numerical simulations of a circuit model which leads to two coupled sine-Gordon equations. The resonant motion of fluxons and antifluxons induces self-pumped current singularities, the *zero-field steps* ( $ZFS \pm n$ ), in the junction dc current-voltage characteristic. They occur at voltages  $V = \pm n \bar{c} \Phi_0 / l$ , where  $\bar{c}$  is the Swihart velocity of light in the barrier of the junction,  $n$  is the number of fluxons/antifluxons, and  $\Phi_0 = h/2e$  is the magnetic flux quantum. Similar phase-locking phenomena have been observed previously by Finnegan and Wahlsten<sup>3</sup> and by Cirillo and Lloyd.<sup>4</sup>

**Experimental Technique**

Experiments were carried out on Nb-Nb<sub>x</sub>O<sub>y</sub>-Pb tunnel junctions which typically had values of  $l = 400 \mu\text{m}$ ,  $w = 20 \mu\text{m}$ ,  $\lambda_J \approx 100\text{--}250 \mu\text{m}$ ,  $\bar{c}/c_{\text{vacuo}} \approx 0.025$  and a critical current density,  $J_c = 0.7 \Delta I_{\text{gap}}/wl$ , of  $3\text{--}16 \text{ A/cm}^2$  (where 70 % of the current increase at the gap,  $\Delta I_{\text{gap}}$ , was used to eliminate effects of the spatial variation of the bias current in the junctions). The sample geometry with two closely spaced JTLs sharing the Pb counter electrode is shown in Fig. 1. Four samples with distance  $d = 35$  or  $75 \mu\text{m}$  between the junctions were investigated. The junction length was chosen to give a resonance frequency  $\bar{c}/2l$  around 10 GHz. As shown in the figure two independent current sources were used and the total voltage across the junctions in series was monitored.

The sample was carefully screened both electrically and

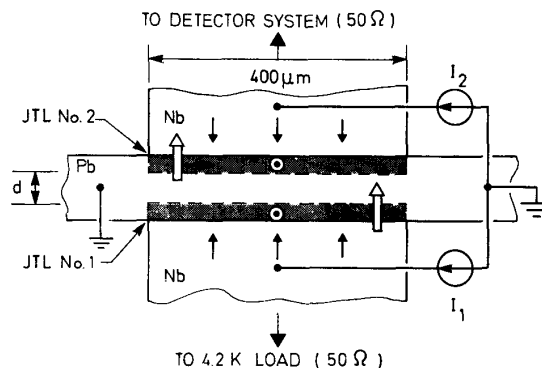


Figure 1. Sample geometry and dc bias arrangement for two Josephson transmission lines (JTLs). The Lorentz force on flux quanta (double arrows) in the junctions is indicated.

magnetically against external noise sources. It was placed in a vacuum can immersed in a pressure-regulated liquid helium bath at 4.2 K. The microwave connection was established by means of a coaxial cable to the  $50\text{-}\Omega$  Nb microstrip line which also formed the bottom electrode of the junctions. The JTL microwave signal was coupled out to a microwave detection system including a spectrum analyzer.

**Experimental Results**

In Fig. 2 we show typical dc I-V curves for the ZFSs of the soliton modes of the junctions individually and in combination. As seen the locking range can be identified from the I-V curves, here shown for the  $(ZFS+1)\text{--}(ZFS+1)$  and the  $(ZFS+1)\text{--}(ZFS-1)$  modes, corresponding to the two current configurations: series aiding ( $\rightarrow\rightarrow$ ) and series opposed ( $\rightarrow\leftarrow$ ).

As seen, the locking range for the series opposed case is particularly clear in this plot since the total voltage  $V_1 + V_2$  equals zero when the JTLs are phase-locked. For two of the samples, A with  $d = 75 \mu\text{m}$  and B with  $d = 35 \mu\text{m}$  the ratio of the locking range for the series opposed configuration was 0.4 for comparable values of  $\Delta \approx 0.30$ , the relative difference in the JTL critical currents:  $\Delta = |I_{c1} - I_{c2}|/\bar{I}_c$  with  $\bar{I}_c = (I_{c1} + I_{c2})/2$ . Sample parameters were  $\bar{I}_c(A) = 1007 \mu\text{A}$ ,  $\bar{I}_c(B) = 259 \mu\text{A}$ ,  $\Delta(A) = 0.31$ ,  $\Delta(B) = 0.30$ ,  $l/\bar{\lambda}_J(A) = 3.4$ , and  $l/\bar{\lambda}_J(B) = 1.7$ , where  $\bar{\lambda}_J = (\lambda_J(A) + \lambda_J(B))/2$ .

The interaction in the form of phase-locking, mixing and linewidth narrowing between the two soliton oscillators was also observed in the detected microwave signals. Figure 3 reproduces some of the microwave spectra from the two

Manuscript received September 24, 1990

junctions both individually and in combination showing phase locking. For two oscillators radiating coherently and in phase the maximum power is  $P_{12} = (\sqrt{P_1} + \sqrt{P_2})^2$  (note that there was a large impedance mismatch between the 50- $\Omega$  microwave circuit and the low impedance JTLs). A coherence ratio  $C$  may be defined as<sup>5</sup>

$$C = \frac{P_{12} - P_1 - P_2}{2\sqrt{P_1 P_2}}$$

where  $C = +1$  corresponds to full coherence with no phase shift between the oscillators. The coherence measurements were made with both the series and the opposed current configuration (see Fig. 4) and showed a surprisingly strong enhancement of the power detected in the coherent state around the middle of the locking range,  $C > 1$  (we note that  $P_1$  and  $P_2$  were measured for the two JTL's separately with the other JTL biased in the zero-voltage state). A narrowing of the linewidth was often observed for the locked oscillators (cf. spectra in Fig. 3). On the top of the ZFSs where the linewidth is less than 50 kHz, which is of technical interest, linewidths down to 4 kHz were observed. A frequency tunability around 1-2 % was typically achieved. Phase locking was also achieved for the (ZFS+2)-(ZFS+1) and (ZFS+2)-(ZFS+2) modes. Measurements on a sample with nearly identical junctions ( $\Delta \approx 0.05$ ) indicated the capability of phase-locking with only one common current source and a frequency tunability up to 7 %. The typical frequency range of phase-locking of 50-200 MHz was about 100 times larger than previously observed for two coupled (soliton) oscillators.<sup>3,4</sup> The linewidth and power were of the same order of magnitude as in these reports ( $\Delta\omega \approx 2\pi \times 10$  kHz,  $P_{12} \approx 20$  pW).

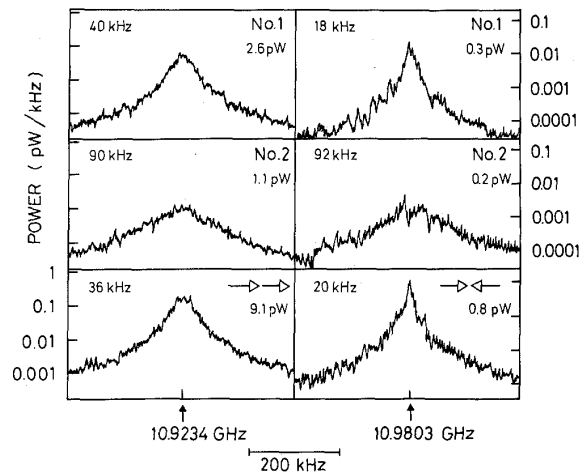


Figure 3. Examples of microwave spectra for two JTLs both individually and in combination. The bottom panels to the left and right are taken at the points "b" and "a" in Fig. 2, respectively. For each spectrum, the FWHM linewidth is indicated in the upper left corner and the total integrated power is given to the right.

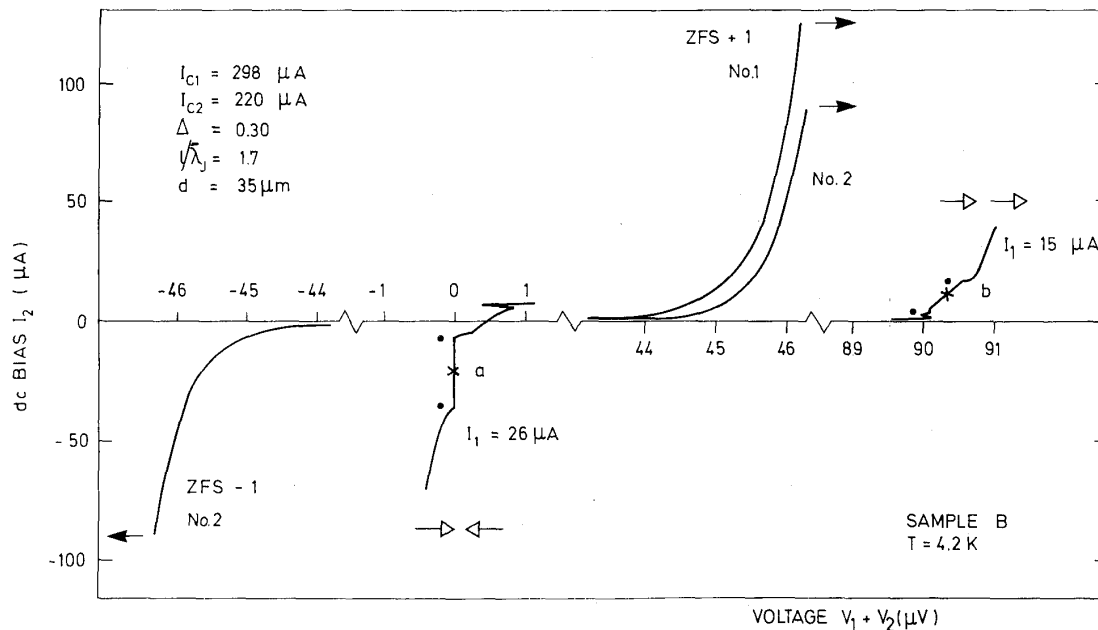


Figure 2. Measured  $I_2 - V_1 + V_2$  characteristics showing the phase locking at dc in a pair of JTLs. The limits of locking ranges are indicated by dots for the (ZFS+1)-(ZFS+1) configuration [series aiding ( $\rightarrow\rightarrow$ )] and for the (ZFS+1)-(ZFS-1) configuration [series opposed ( $\rightarrow\leftarrow$ )].

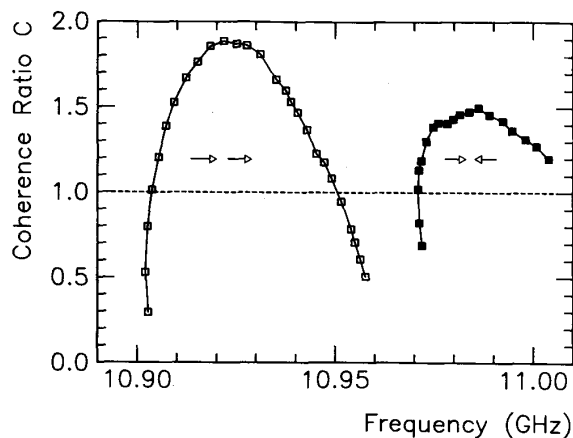


Figure 4. The coherence ratio  $C$  for sample B measured over the locking ranges marked by dots in Fig. 2.

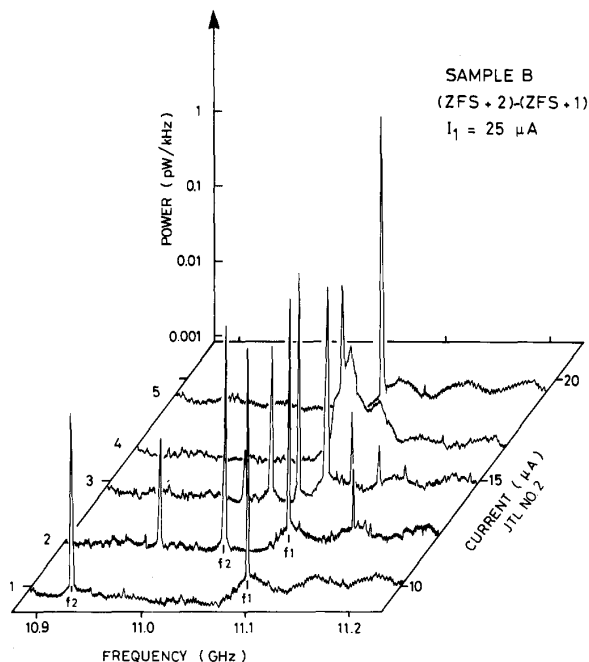


Figure 5. A typical series of microwave spectra measured for various bias conditions around the edge of a phase-locked region ( $I_1$  is constant at 25  $\mu\text{A}$  while  $I_2$  is varied). Trace 1 shows the two soliton oscillators running independently while trace 5 shows the coherent state just inside the locking range. Frequency pulling and mixing products appear for intermediate bias values (traces 2 and 3). Quasiperiodic, "noisy" spectra are observed just outside the locking region (see trace 4).

### Theoretical Model

When modelling the coupled soliton oscillators we represent the interaction in the form of a distributed mutual inductance between two resistor-capacitor shunted junctions (RCSJ model). This leads to two coupled perturbed sine-Gordon equations.<sup>6</sup> A similar approach was taken in Ref. 7 to model two JTLs vertically stacked one above the other and sharing a common center electrode, although in that system only one JTL was operating as a fluxon oscillator (the second served as a radiation detector), and hence no locking phenomena were involved. In Refs. 8 and 9 the interactions between a pair of coupled JTLs was modelled in this manner, although the authors did not explicitly consider phase locking. Since our sample geometry places the JTLs side by side, the magnitude of the mutual inductance  $M$  (per unit length) will be much less than for a geometry of two stacked JTLs<sup>9</sup>. Nevertheless, we find that locking arises even for small values of  $M/L$ <sup>6</sup> ( $L$  is the inductance of the JTL per unit length). An elementary calculation on normal conductors gives approximately  $M/L = sw/(\pi d(d+w))$ , where  $s$  is the sum of the dielectric thickness and the London penetration depths of the two electrodes. For the Nb/NbOx/Pb junctions,  $s=133$  nm. The value of  $M/L$  from this estimate is an order of magnitude smaller than the values found by comparing experimental data with simulations.

Using this inductive coupling model, we have carried out extensive numerical simulations of the phase-locking phenomena in two coupled JTLs. The locking range was investigated for fixed (typical) values of the junction parameters  $l/\lambda_J = 3$ ,  $\alpha = 0.05$ ,  $\beta = 0.02$  and  $\Delta = 0.30$  while  $M/L$  was varied over the range 0.01 to 0.03. The sign of  $M/L$  was chosen in accordance with the interaction between two magnetic dipoles. The loss parameters  $\alpha$  and  $\beta$  are determined by the quasiparticle shunt and surface losses, respectively. Good qualitative agreement between experiment and model was found in the I-V curves<sup>6</sup> as well as in the high frequency spectra around the locking range, where strong frequency pulling and mixing phenomena were seen. We conclude that the model predicts most of the observed phase-locking behavior, although no effective gain was seen in the simulations. The present model could be improved by including a nonlinear loss term and a spatial variation of the coupling parameter and of the bias current (peaked at the edges of the thin-films).

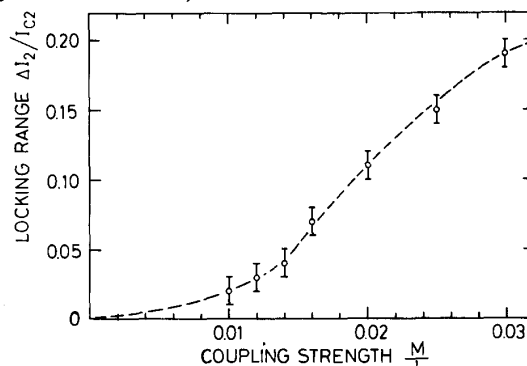


Figure 6. Results of numerical simulations of the locking range  $\Delta I_2/I_{c2}$  vs. the coupling strength  $M/L$ . The parameters were  $I_1/I_{c1} = 0.4$ ,  $\Delta = 0.3$ ,  $l/\lambda_J = 3.0$ ,  $\alpha = 0.05$ , and  $\beta = 0.02$ .

### Conclusions

We have demonstrated strong phase-locking phenomena between two closely spaced fluxon oscillators based on intermediate length Josephson transmission lines. Phase-locking was seen both in the dc I-V characteristics and in the high frequency radiation. The development of coherent microwave sources consisting of many Josephson junctions is of technological interest.<sup>10</sup> We have modelled the system as two inductively coupled Josephson transmission lines. The resulting set of two coupled perturbed sine-Gordon equations has been solved numerically. Agreement between numerical results and experimental observations is generally good, although the observed effective gain ( $C > 1$ ) was not seen in our simulations.

### Acknowledgements

We thank M.R. Samuelsen, A. Davidson, L.E. Guerrero, C. J. Lobb, M. Salerno, M. Devoret, and N.F. Pedersen for fruitful discussions, G. Friis Eriksen for technical assistance and I. Rasmussen and S. Hjorth for the sample fabrication. This work was partly supported by the Danish Natural Science Research Council, The Danish Research Academy, the Højgaard Foundation and the Natural Sciences and Engineering Research Council of Canada.

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